Boundary Integral Equation Approach to Multi-Mode Y-Matrix Characterization of Multi-Ridged Sections in Circular Waveguide

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Abstract

Aiming at an accurate and effective CAD procedure for **Circular Waveguide** (CW) **dual-mode filters** without tuning screws, the **Boundary Integral Equation Method** (BIEM) is applied to TM/TE mode analysis of **Multi-Ridge Circular Waveguide** (MRCW) and evaluation of CW-MRCW mode coupling coefficients to obtain the **Multi-Mode Network (Y-Matrix) Representation** (MMNR) of CW-MRCW transitions. A novel **Multiple Eigenvalue Search Algorithm** assures completeness of the modal spectrum up to large mode indices (> 100) in the presence of accidental degeneracies and clusters of eigenvalues.

Introduction

CW dual-Mode filters offer high Q-factors at reduced size and weight, but the high expenditure for manual tuning is a major drawback. Replacement of tuning screws by exactly premachined sections of MRCW (Fig. 1) was therefore proposed in [1]. To make it feasible, accurate design tools are required: essentially a rigorous *electromagnetic field* (EMF) analysis embedded in a filter optimization procedure. A rigorous approach to this problem is the well known MMNR of discontinuities between cascaded waveguide sections (e.g. [2]). By decomposing the problem such as to leave the EMF analysis part (determination of cut-off frequencies and modal coupling coefficients) frequency independent and only a relatively simple multi-mode transmission line network analysis frequency dependent, it is also very efficient.

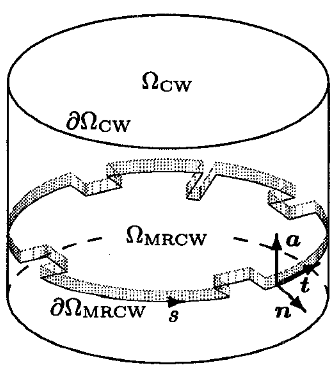

Fig. 1: *Short section of MRCW replaces tuning screws in a CW dual-mode filter.*

As opposed to some classical applications (e.g. [3]), MRCW cut-off frequencies and CW-MRCW modal coupling coefficients are accessible only by numerical methods. This problem has been subject of a considerable number of investigations in the recent past [1, 4, 5, 6, 7]. Apart from accuracy and efficiency requirements, completeness of the computed spectrum up to large mode indices (typically several hundreds), for all geometrical parameters encountered within an optimization loop, is a key problem in this context. Up to now, there is, to the authors knowledge, only a single very recent publication [8] which gives results for more than a few low order modes in MRCW.

Combining the MMNR with the BIEM [9, 10] offers several distinct advantages. The 3-d EMF problem is reduced to a set of 1-d eigenvalue problems for plain functions. Coupling coefficients are efficiently obtained as 1-d inner products over these functions. Also, in contrast to numerical methods which dictate allowed ridge shapes [7, 8], exact parametric representation of an arbitrarily curved waveguide contour is possible. Furthermore, field singularities can easily be accounted for.

Analytical Formulation

The fields within a waveguide cross-section $\Omega \subset \mathbb{R}^2$ (Fig. 1) can be derived from TM and TE Hertz potentials $\boldsymbol{a}\psi'_n$ and $\boldsymbol{a}\psi''_n$, respectively, where $\psi'_n, \psi''_n : \Omega \to \mathbb{R}$ are eigenfunctions of the Helmholtz equation with eigenvalues $h'_n \in \mathbb{R}_+$ (TM) and $h''_n \in \mathbb{R}_+$ (TE). The BIEM reduces the 2-d eigenvalue problems to 1-d eigenvalue problems for the *traces*

$$u : \mathrm{I} \to \mathbb{R}, \quad s \mapsto \psi(\boldsymbol{r}(s)) \qquad (1)$$

$$\text{and} \qquad v : \mathrm{I} \to \mathbb{R}, \quad s \mapsto \boldsymbol{n_r}\,\mathbf{grad}_{\boldsymbol{r}}\,\psi(\boldsymbol{r}(s)) \qquad (2)$$

of ψ'_n and ψ''_n along the waveguide contour $\partial\Omega$ with the parameterization $\boldsymbol{r} : \mathrm{I} \to \partial\Omega, s \mapsto \boldsymbol{r}(s)$. v'_n and u''_n, which correspond to TM mode axial and TE mode transverse tangential surface current densities, respectively, are obtained as eigenfunctions of the BIEs

$$\mathbf{G}[v'](t) := \int\limits_{\mathrm{I}\setminus\{t\}} g_{h'}(t,s)v'(s)\,\mathrm{d}s = 0 \qquad (3)$$

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and

$$\mathbf{K}[u''](t) := \int_{\mathrm{I}} u''(s)\Big(k_{h''}(t,s) - k_0(t,s)\Big)\,\mathrm{d}s + \int_{\mathrm{I}\setminus\{t\}} \big(u''(s) - u''(t)\big)\,k_0(t,s)\,\mathrm{d}s = 0. \quad (4)$$

With the abbreviations $\boldsymbol{p} := \boldsymbol{r}(t)$ and $\boldsymbol{q} := \boldsymbol{r}(s)$, the kernels are given by $g_h(t,s) := K_0(\mathrm{j}\,h\,\|\boldsymbol{p}-\boldsymbol{q}\|)$, which for $h = 0$ becomes $g_0(t,s) = -\ln(\|\boldsymbol{p}-\boldsymbol{q}\|)$, and its normal derivative $k_h(t,s) := \boldsymbol{n}(s)\,\mathbf{grad}_{\boldsymbol{q}}\,g_h(t,s)$. The decomposition of the integral in (4) assures uniform convergence of each term as $\boldsymbol{r}(t)$ approaches a singular boundary point, which is required for numerical evaluation of the integrals [9].

For evaluation of coupling coefficients only the above traces v'_n and u''_n for MRCW and in addition the analytically accessible traces u'_m and v''_m of TM and TE modes in CW with respect to the MRCW contour and their eigenvalues h'_m and h''_m are required. With the understanding that subscript m refers to CW and subscript n to MRCW quantities, respectively, the non-vanishing coupling coefficients are obtained as

$$c_{m,n}^{\mathrm{TM,TM}} = \frac{h_n'^{\,2}}{h_n'^{\,2} - h_m'^{\,2}} \int_{\mathrm{I}} u'_m(s) v'_n(s)\,\mathrm{d}s, \quad (5)$$

$$c_{m,n}^{\mathrm{TM,TE}} = \int_{\mathrm{I}} u'_m(s) \frac{\mathrm{d}}{\mathrm{d}s}\, u''_n(s)\,\mathrm{d}s, \quad (6)$$

and

$$c_{m,n}^{\mathrm{TE,TE}} = \frac{h_m''^{\,2}}{h_m''^{\,2} - h_n''^{\,2}} \int_{\mathrm{I}} v''_m(s) u''_n(s)\,\mathrm{d}s. \quad (7)$$

The integrals extend over $\mathrm{I} = \boldsymbol{r}^{-1}(\partial\Omega_{\mathrm{MRCW}}) \subset \mathbb{R}$. The usual definition would require that all the trace functions in (5)-(7) are normalized such that each mode carries a power of unity. For the MRCW modes this normalization is not easily obtained with the present approach. In a cascaded structure, however, mode normalization is of significance only in the reference planes. If these allow for analytical treatment, as in the case of CW, lack of a normalization procedure presents no problems.

Numerical Procedure

For approximate numerical solution of (3) and (4) the trace functions v'_n and u''_n are expanded into N quadratic B-splines over a partition of the Interval I which maps onto $\partial\Omega$ with approx. 4 elements per transverse wavelength $\lambda_{\min}$ of the highest desired mode (Fig. 2). A priori estimates of $\lambda_{\min}$ are obtained from the asymptotical distributions of CW eigenvalues. To solve for the first 100 TE or TM modes requires no more than approx. 100 expansion functions. The small

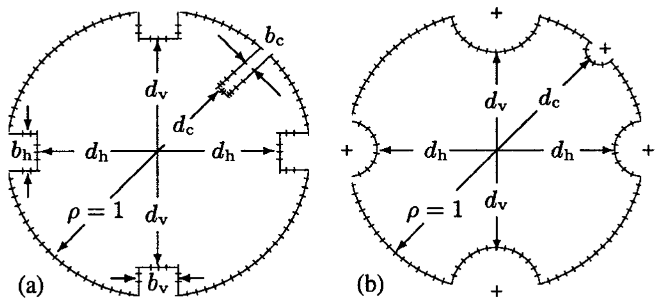

Fig. 2: *Geometrical parameters and boundary partition for MRCW with (a) rectangular and (b) circular ridge shape.*

number reflects the absence of orthogonality relations between trace functions for different eigenvalues. While u'' (TE mode tangential surface current density) is regular and well represented with B-splines, v' (TM mode axial surface current density) exposes weak singularities about singular boundary points. Apart from a non-equidistant partition (Fig. 2a) edge terms with the precomputed asymptotically exact singular behaviour are therefore added. Numerical evaluation of the integrals comprises an adaptive per element coordinate transform to remove kernel and edge singularities followed by a modified Romberg scheme which terminates at a specified relative error (typically 10^{-7}). This approach decouples the integration error from the number of expansion functions and such assures an accurate representation of curved boundary segments. Discretization of the residual of the BIEs relies on the *Method of Least Squares with Intermediate Projection* with approx. $2N$ weighting functions, an approach which is necessary to exclude the occurrence of spurious solutions [11]. The final (overdetermined) homogeneous matrix equations are solved by *singular value decomposition* (SVD). For given trial value of h (i.e. either h' or h''), the smallest attainable residual is just the smallest singular value $\sigma_N(h)$. Near zeroes of $\sigma_N(h)$ indicate that the corresponding singular vector $C_N(h)$ approximates the coefficients of expansion of an eigenfunction.

Multiple eigenvalue search algorithm

The key problem for the present application is completeness of the spectrum up to large mode indices (hundreds of TE and TM modes, each) for any set of geometry parameters which may be encountered within an optimization loop. Accidental degeneracies and clustering of eigenvalues, however, present a severe problem for all methods which include a search procedure for detection of eigenvalues. Approaches which rely on tracing the minimally attainable residual $\sigma_N(h)$ are bound to fail, however small the sampling interval may be chosen (Fig. 3a).

The solution to this problem was accomplished by a novel *Multiple Eigenvalue Search Algorithm* (MESA). It is based

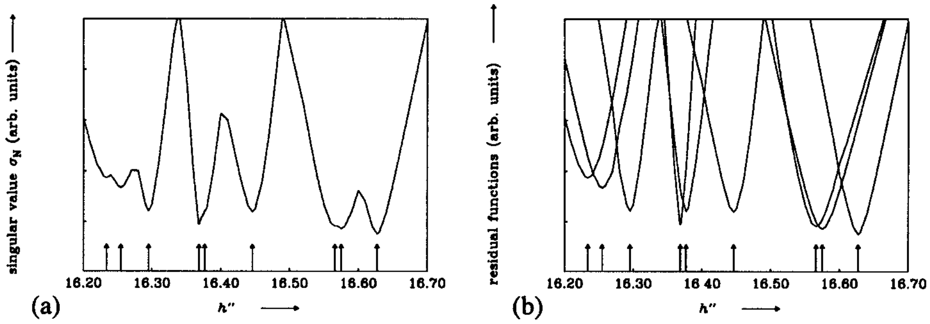

Fig. 3: *Typical example for (a) minimum residual $\sigma_N(h'')$ of (4) as usually considered, (b) set of residual functions $r_j(h'')$ as introduced in the new algorithm (MESA) which allows for reliable detection of (nearly) degenerate modes. Arrows mark eigenvalues.*

on (i) the analyticity of all quantities with respect to h, (ii) the orthogonality between degenerate trace functions and (iii) the fact that the SVD yields, for each trial value of h, a set of mutually orthogonal candidate solutions (the singular vectors $C_i(h)$) ordered in sequence of decreasing residual (the singular values $\sigma_i(h)$). As a consequence *degenerate* solutions identify as near zeroes of the residuals $\sigma_N(h), \sigma_{N-1}(h), \ldots$ of two or more *orthogonal* candidate solutions $C_N, C_{N-1}(h), \ldots$. Due to analyticity in h, closely neighbored eigenvalues still correspond to almost orthogonal traces and singular vectors. Hence, by introducing an index transformation $P : \{1, \ldots N\} \rightarrow \{1, \ldots, N\}$, based on comparing inner products between singular vectors $C_i(h)$ and $C_j(h+\Delta h)$, it is possible to define residual functions $r_i(h) := \sigma_{P(i)}(h)$ which are bound to and evolve together with a specific candidate solution (Fig. 3b). By examination of the smallest (currently 6) of these residual functions identification of very closely neighbored eigenvalues is put on the same footage as that of degenerate eigenvalues. The approach allows for reliable detection of all eigenvalues with little numerical overhead. It even improves overall efficiency by allowing for a larger sampling interval Δh.

Application Examples

The method has proven successful in determination of large numbers of TE and TM modes and evaluation of coupling constants for several MRCW configurations with rectangular (Fig. 2a), circular (Fig. 2b) and also trapezoidal ridges. Only a few typical results can be given here. Fig. 4 displays the dependence of TM and TE eigenvalues on the penetration of the coupling ridge for a MRCW configuration which was previously considered in [1]. Fig. 5 shows the surface current density along $\partial\Omega$ for two randomly selected TM and TE modes in this configuration. The TM mode edge singularities are clearly a disadvantage because they require for a large number of CW modes within the MMNR, and also add to ohmic losses. They can be avoided with a smooth ridge shape and it may be counted as one of the advantages of the present approach that there are basically no restrictions on the shape of the MRCW contour. Results for circular ridge shape (Fig. 2b) are given in Fig. 6 with the penetration of the horizontal pair of tuning ridges as a parameter. Fig. 7 illustrates the absence of edge singularities.

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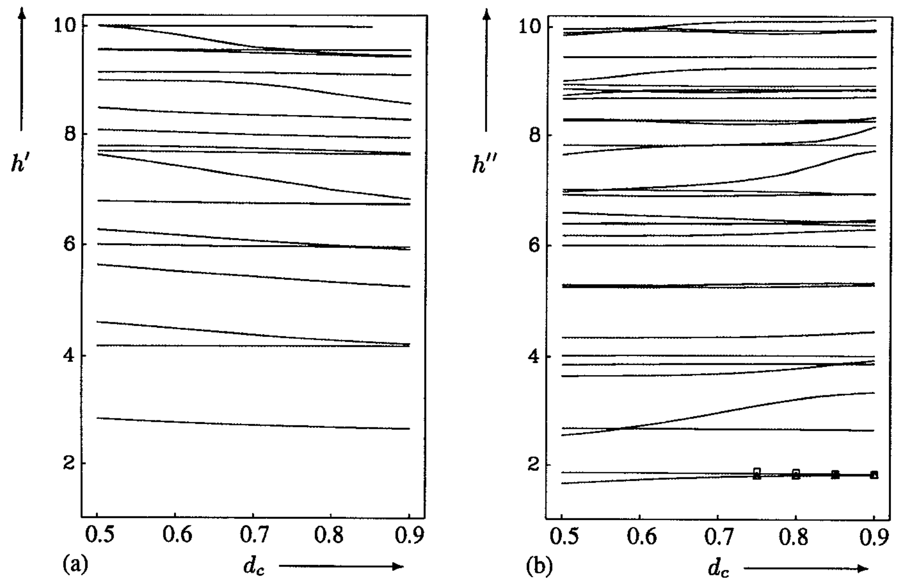
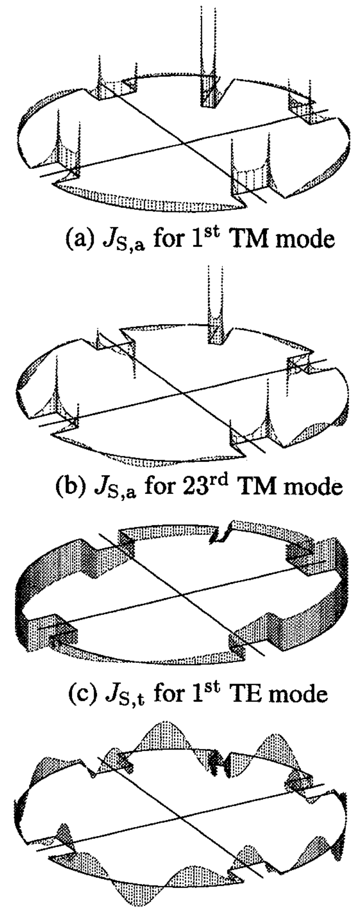


Fig. 4: *Influence of coupling ridge penetration* $(1 - d_c)$ *on (a) the first 17 TM and (b) the first 30 TE eigenvalues for the MRCW of Fig. 2a with* $d_h = d_v = 0.8$, $b_h = b_v = 0.26105$ *and* $b_c = 0.08724$. *Symbols refer to data from [1].*

Fig. 5: *Illustration of (a,b) axial and (c,d) transverse tangential surface current densities along the MRCW contour of Fig. 2a with parameters as given in the caption of Fig. 4 and* $d_c = 0.55$ *in case (a),* $d_c = 0.75$ *in case (b) and* $d_c = 0.8$ *in cases (c) and (d).*



(a) $J_{S,a}$ for 1st TM mode

(b) $J_{S,a}$ for 23rd TM mode

(c) $J_{S,t}$ for 1st TE mode

(d) $J_{S,t}$ for 53rd TE mode

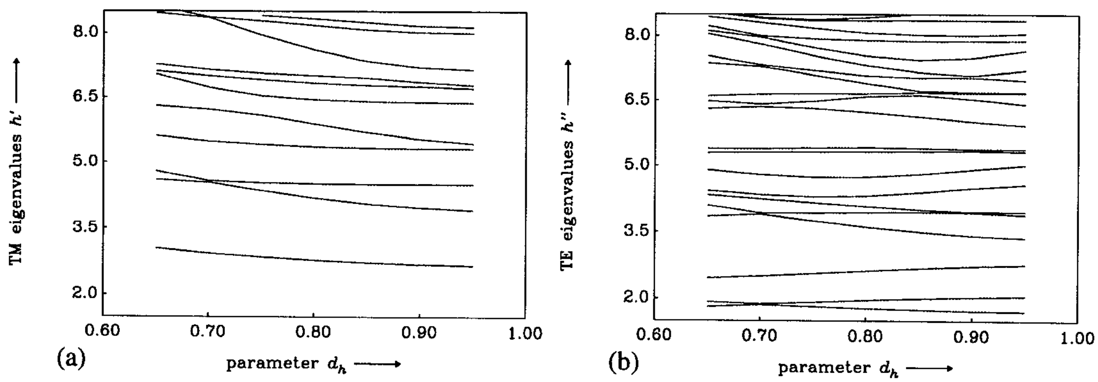
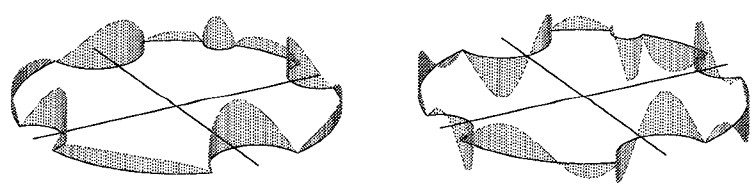


Fig. 6: *Dependence on the parameter* d_h *of (a) the first 11 TM and (b) the first 21 TE eigenvalues for the MRCW of Fig. 2b with* $d_v = 0.7$ *and* $d_c = 0.9$ *fixed.*



(a) $J_{S,a}$ for 1st TM mode

(b) $J_{S,a}$ for 16th TM mode

Fig. 7: *Avoidance of TM-mode singularities due to smooth ridge shape for MRCW of Fig. 2b with* $d_h = 0.8$, $d_v = 0.7$ *and* $d_c = 0.9$.